SHAPE OPTIMIZATION PROCEDURES OF UNDERGROUND TUNNEL EXCAVATIONS

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underground excavation, shape optimization, linear elasticity, energy of volumetric strain.

INTRODUCTION

Conventional tunnelling methods were based mainly on the engineers' experience. In this case excavation method and support construction technology were strictly associated with final shape, which was usually a horseshoe shape. Modern tunnelling technologies can be divided into two groups: with determined shape, e.g. TBM methods and with undetermined shape. For example widely spread NATM gives the possibility to shape the cross-section without any strong restriction. In this context the shape optimization procedures become very important topic in contemporary tunnel design. Optimization gives the possibility to lower the costs and to eliminate the influence of construction on the surrounding rock mass. This paper concerns different procedures for excavation shape optimization techniques.

The arrangement of the paper is as follows. Firstly, the consideration of elliptical shapes developed by Sałustowicz is presented. Then, an application of the evolutionary structural optimization (ESO) procedure for underground excavation shape is outlined. The results constitute Sałustowicz's assumption that the optimal are elliptical shapes with appropriate ratio of semiaxes depending on in-situ stresses. Subsequently the formulation of a condition based on the energy of volumetric strain is presented. The numerical simulations concerning the tunnel with a support are presented as a verification of the condition. Final conclusions end the paper.

CLASSICAL APPROACH

The first mention of shape optimization appears in literature in 1960s (Sałustowicz, 1968). Sałustowicz has based his consideration on the solution of elastic rectangular plate with an elliptical opening in the centre. The plate has been assumed to be in plain stress and loaded uniformly in vertical direction with intensity of p_z and horizontally with intensity of p_x. A scheme of the considered plate is shown in Fig. 1. According to the known analytical solution of the problem (Huber, 1950) the extreme stresses occur in the sidewall and/or in arch vertex. Values of these extreme stresses depend on the pressures p_x, p_z and on the ratio of semiaxes of the ellipse a/b. The extreme stresses values can be expressed by the following formulas:

(1) $\sigma_x = -p_z + p_x\left(1 + 2\frac{b}{a}\right)$

(2) $\sigma_z = p_z\left(1 + 2\frac{a}{b}\right) - p_x$

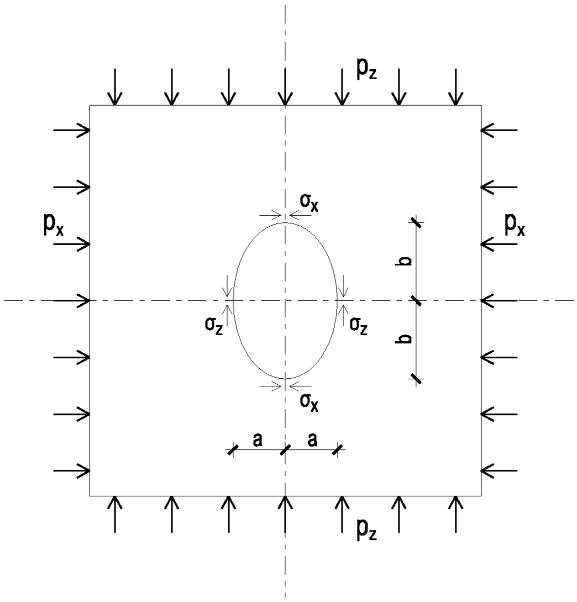


Fig. 1. A rectangular plate in plain stress with elliptical opening in the centre

For given values of pressures p_x and p_z it is possible to determine optimal ratio of the ellipse semiaxes that minimize the extreme stress in the plate. Sałustowicz has considered different values of semiaxes ratios $a/b=m$. He has stated that the optimal value of m depend on the ratio of pressures p_x/p_z and is given by the relation:

$$(3)\ m_{opt} = \left(\frac{a}{b}\right)_{opt} = \frac{p_x}{p_z}$$

Furthermore, for the optimal ratio of semiaxes m_{opt}, stresses in the sidewall and in arch vertex are:

$$(4)\ \sigma_x = \sigma_z = p_x + p_z$$

The considerations above are devoted for underground excavations without any support. Sałustowicz has given also the other condition for optimal shape of supported excavation. He has concluded that the best cross-section of excavation is an ellipse with quotient of semiaxes equal to the square root of the ratio p_x/p_z in accordance with formula:

$$(5)\ m_{opt} = \left(\frac{a}{b}\right)_{opt} = \sqrt{\frac{p_x}{p_z}}\ .$$

EVOLUTIONARY STRUCTURAL OPTIMIZATION (ESO)

A few years ago the evolutionary structural optimization (ESO) was employed to find optimal shapes of an unsupported tunnel cross-section (Ren G., 2005). The procedure of ESO was originally developed in the 1990s (Xie Y. M., 1993). The ESO is an iterative proccess. In each iteration a portion of inefficient material is removed from the domain. Each iteration consists of following steps:

- for given boundary condition and constraints finite element analysis (FEA) is used to calculate stress distribution over the domain,
- a list of finite elements is sorted in descendig order of the level of efficiency which is defined by appropriate criterion based on stresses,

- a portion of finite elements from the beginning of the list is treated as inefficient material and is removed from the domain.

The number of elements removed in one iteration step is detrmined by two parameters VR (volume removal rate) and RR (rejection ratio). First parameter defines how many elements can be removed in one iteration. It is expressed as a fraction (percentage) of a total material volume. The second parameter sets the treshold of stress level. Typical values of VR and RR are about 5%.

As it was mentioned earlier the ESO procedure was employed to perform opimization of underground excavation shape. In the following the results obtained by Ren G. et al (Ren G., 2005) are presented. The finite element regular mesh of 70 x 70 elements, boundary conditions as well as initial void are presented in Fig. 2. Note that the starting void is required to perturb the uniform initial stress distribution. In this case it is a square of 5 x 5 elements. In addition, the first invariant of stress tensor is used as an efficiency criterion.

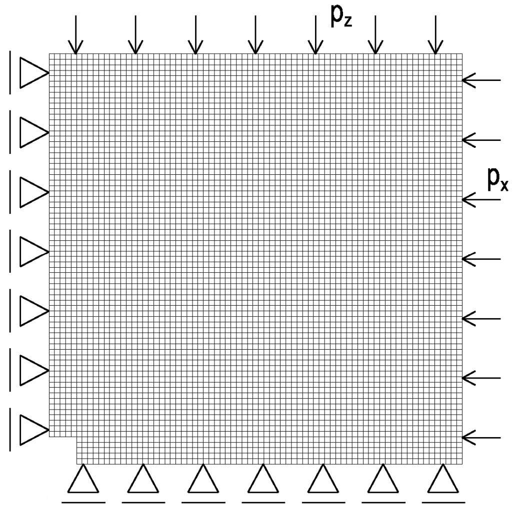
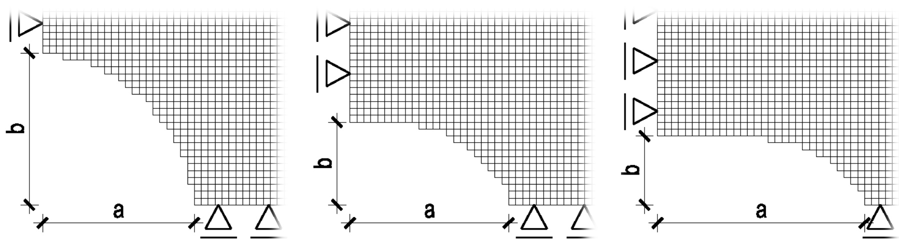


Fig. 2. Initial finite element mesh a quarter of a space around the excavation with the initial void

The procedure was performed for three different values of p_x/p_z ratios. The shapes obtained after 70 iterations, as the results of the ESO procedure, are presented in Fig. 3.



a) b) c)

Fig. 3. Results of ESO procedure after 70 iterations for different p_x/p_z ratios:
a) p_x/p_z =1; b) p_x/p_z =2; c) p_x/p_z=3

The results support the Sałustowicz's statement, i.e. the optimal shape of a tunnel cross-section is an ellipse, which semiaxes satisfy the equation (3).

The cited work (Ren G., 2005) covers wider range of problems not only optimization of tunnel cross-section but also two tunnels intersection as well as 3D problem of a closed cavern. Nevertheless it is not a topic of this work and so these problems are not reflected in this paper.

FORMULATION OF OPTIMIZATION CONDITIONS

A basic function of support is to ensure the stability of excavation. In tunnels at great depths, where the deformation pressure is the dominant load, this function is performed by opposing of "tightening" of the rock-mass. The "tightening" effect can be measured by integral of displacements expressed as:

$$(6)\quad \Delta V = \int_S \vec{\mathbf{u}}\vec{\mathbf{n}}\, dS \,.$$

The quantities used in eq. (6) are explained in Fig. 4.

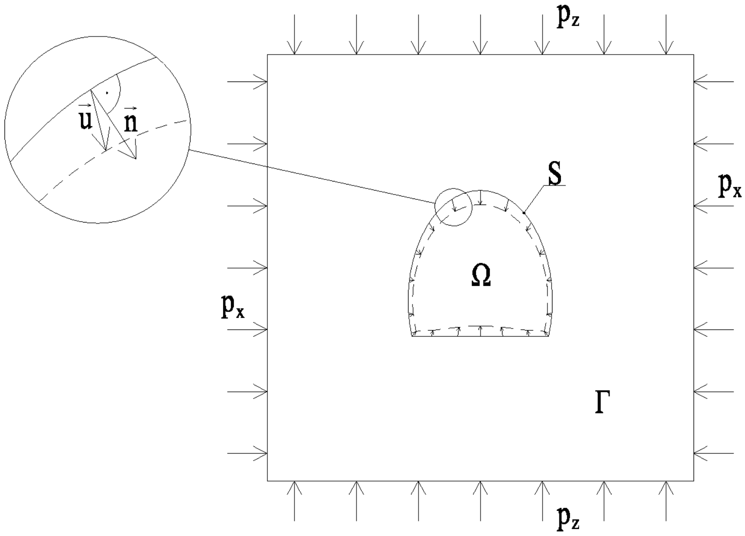


Fig. 4. Adopted designation: *Γ*, *Ω* rock mass and excavation area, respectively; *S* – excavation contour, $\vec{\mathbf{u}}$, $\vec{\mathbf{n}}$ displacement and unit normal vector, respectively

It is obvious that support effort strongly depends on value of Δ*V*. That means an optimal shape of excavation is the one that corresponds to the minimum value of Δ*V*.
Quantity (6) has some disadvantages. For example for some rectangular excavation shapes the value of (6) can be close to 0 in spite of strong deflection of excavation. That is because it is not positive defined. A better objective function is the energy of volumetric strain in excavation core (7). Such quantity is also the measure of "tightening", furthermore it is positively defined and is easier to interpret. In the equation:

$$(7)\quad E_0 = \frac{1}{2} K \int_\Omega \left(\varepsilon_x + \varepsilon_z\right)^2 d\Omega$$

K denotes the bulk modulus,
ε_x, ε_y are the values of normal strain horizontal and vertical, respectively.
Optimization procedures restricted to the ellipsoidal shapes of excavation using (6) and (7) as objective function should be identical. Verification of this hypothesis is presented in the next section.

NUMERICAL EXAMPLE

To verify optimization conditions formulated in previous section proper numerical calculations has been carried out (Różański, 2009). Commercially available finite element code – FlexPDE has been used to solve the boundary value problems. Three series of calculation has been performed for different values of horizontal to vertical pressure ratios, i.e. p_x/p_z=1, p_x/p_z =1/4 and p_x/p_z =1/9. The aim of calculation in each of three cases was to find an optimal shape in sense of minimal energy of volumetric strain E_0 and also minimal “tightening” ΔV. Only elliptical shapes of fixed value of the area has been considered. It follows that the only variable of a shape is the ratio of semiaxes lengths $m=a/b$. Linear elastic material has been assumed. Material constants for rock mass region Γ are the Young’s modulus E=7e10 Pa and the Poisson’s ratio ν=0,26. The core region Ω is assumed not to be a simple void but an elastic material with negligible stiffness. The Young’s modulus for the core region Ω is 10^{10} times smaller than for the region of rock mass Γ. Such approach makes possible to calculate the energy in the core of excavation and does not disrupt the results of calculations. The plots of the energy of volumetric strain E_0 and “tightening” ΔV versus m for three cases of considered in-situ stresses ratios are presented in Fig. 5.

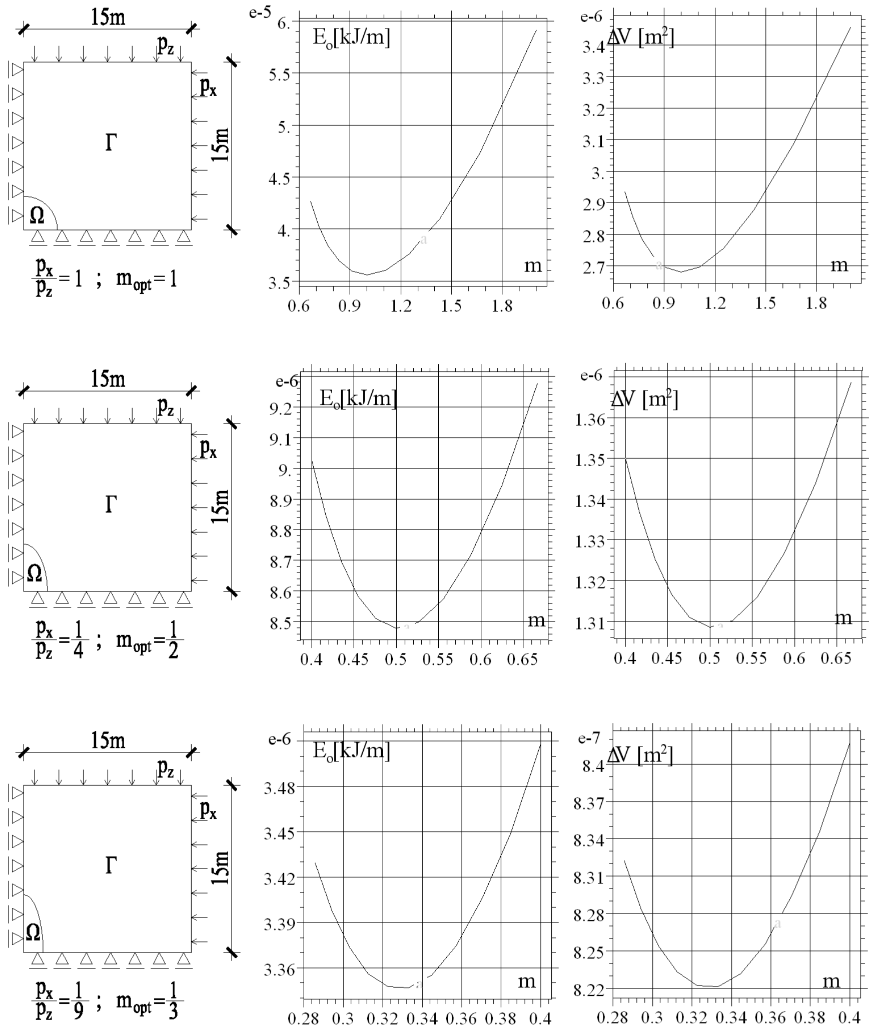


Fig. 5. Plots of E_0 and ΔV versus m for three different p_x/p_z values

The results presented above confirm hypothesis that both integrated displacement (6) as well as volumetric strain energy (7) can be used as objective function for optimization of excavation shape. Optimal semiaxes ratios corresponding to the minimum E_0 and ΔV are identical and they satisfy the equation (5) proposed by Sałustowicz.

Since excavation support hasn’t been modelled in above calculation, quantities (6) and (7) can be treated as measures of excavation support effort only potentially. Proposed objective function as well as obtained formula for optimal ellipsoidal shape (5) need to be verified.

VERIFICATION OF THE OPTIMIZATION CONDITIONS

In order to verify obtained results a new series of boundary value problems with permanent support has been performed (Kawa M., 2011). As previously, linear elastic material is assumed for rock mass as well as for support material. Parameters are assumed E_r=7e7Pa, v_r=0,3 for rock mass and E_s=7e9Pa, v_s=0,3 for support. So the stiffness of support is 100 times greater than the stiffness of surrounding rock. The tunnel interior is a void. Support has been modelled as region with constant width (thickness) t=0,5 m and with centre line shaped elliptic. Due to the symmetry only the quarter of the domain is considered. A series of calculation has been carried out for five different values of horizontal to vertical pressure ratios, namely: p_x/p_z =0,5; p_x/p_z =0,75; p_x/p_z =1,0; p_x/p_z =1,25 and p_x/p_z =1,5. In each series variation of the excavation shape is taken into account. As previously class of considered shapes has been limited to elliptical shapes and constant area of the excavation is assumed. Static scheme of the problem is shown in Fig. 6.

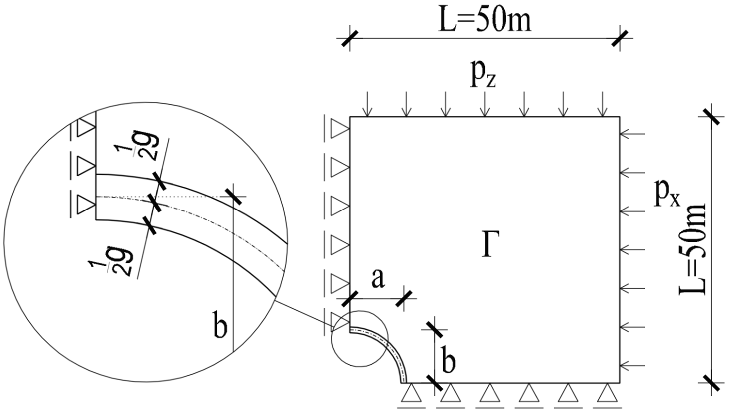


Fig. 6. Static scheme of the problem under consideration

Additionally it has been assumed that lowest effort of support takes place when maximal (absolute) normal stress in support reaches minimum. It has been observed that maximal values of normal stresses are always present in arch vertex or in sidewall of the cross-section of tunnel support either on its internal or external side. Maximum normal stress in the support is equal to maximum of the four stresses. Adopted designation of the stresses is as follows: σ_{xi}, σ_{xe} are horizontal normal stress on internal and external side of support in arch vertex; σ_{zi}, σ_{ze} are vertical normal stress on internal and external side of support in sidewall.

Example results for p_x/p_z =1,5 are presented in Fig. 7. On the left plot of "tightening" ΔV versus semiaxes ratio m, on the right stresses σ_{xi}, σ_{xe}, σ_{zi}, σ_{ze} versus m. Contour S has been assumed as internal side of support. On both plots dashed line refers to the minimum of considered quantity.

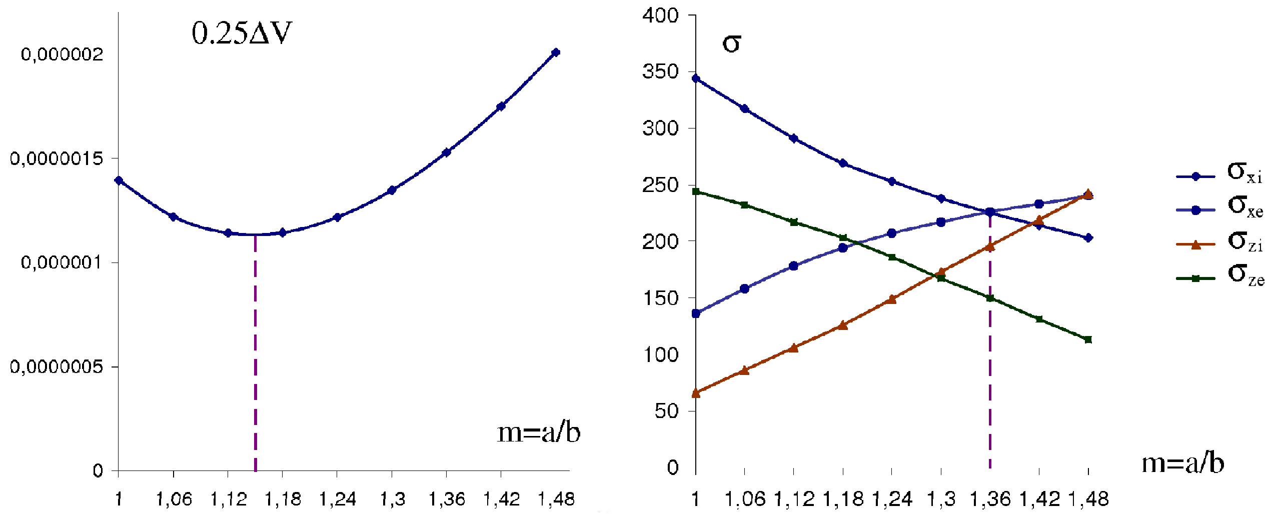


Fig. 7. Example results for p_x/p_z =1,5

Results for all five relations p_x/p_z are summarized in Fig. 8. The horizontal axis on the plots is p_x/p_z, vertical axis is optimal semiaxes ratio according to different conditions: m_{opt}^{σ} corresponds to minimal value of overall normal stress over the domain, $m_{opt}^{\Delta V}$ corresponds to minimal value of ΔV, m_{opt}^{Eo} corresponds to minimal value of the energy of volumetric strain in the support. The value of m_{opt} according to eq. (5) is also presented.

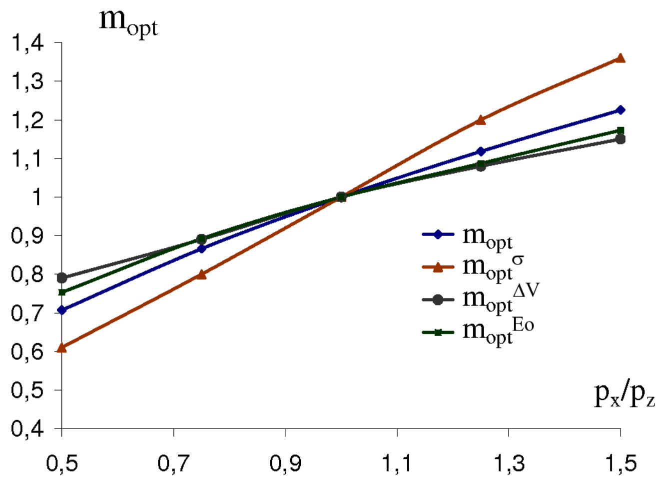


Fig. 8. Results for all five relations p_x/p_z

It is easy to notice that all presented curves pass through the point (1,1) and its course is similar to the power function:

$$(8)\quad m_{opt} = \left(\frac{a}{b}\right)_{opt} = \left(\frac{p_x}{p_z}\right)^a$$

Plots of $m_{opt}^{\Delta V}$ and m_{opt}^{Eo} have almost identical course. These curves are sloped less than the curve describing Sałustowicz's condition. That corresponds to power a in eq. (8) smaller than 0,5 proposed by Sałustowicz. The course of m_{opt}^{σ} plot corresponding to the minimum of maximal normal stress in support can be described with use of power a greater than 0,5. Summing up the Sałustowicz's condition gives intermediate results.

CONCLUSIONS

Three different approaches for tunnel shape optimization procedures were presented. The classical approach is based on an analytical solution of a plate with elliptic opening. The ESO procedure originated in 1990s confirms that the elliptic shape is optimal for simple unsupported tunnels. Both classical solutions and the ESO determine optimal semiaxes ratio equal to the ratio of in-situ stresses. In 2009 a procedure based on the energy of volumetric strain in a core was proposed by the authors. The procedure is formulated to optimize the shape of a tunnel with a support. This problem is more complicated because of the difficulties in choosing of proper objective function for optimization. Taking the maximal stress in support or average displacement of support as the objective function gives different results. The procedure based on the energy of volumetric strain in a core gives the intermediate results. That was verified by numerical examples concerning the tunnel with a support taken into account.

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